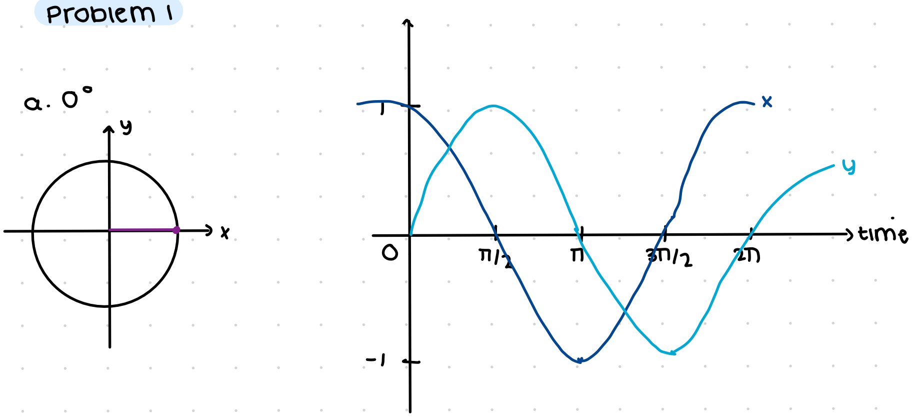
JIGSAW-1E

2

Assuming anti-clockwise rotation... and also, you are assuming that time = angle/omega

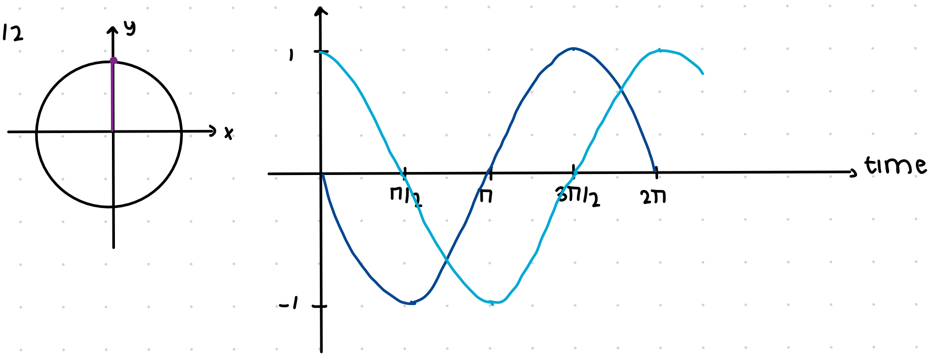
Problem 1

a. $0°$



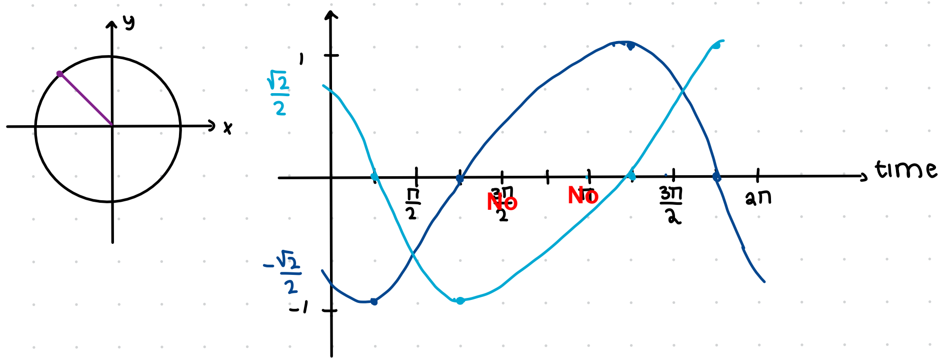
- x-component $= r\cos(\omega t + 0) = r\cos(\omega t)$
- y-component $= r\sin(\omega t + 0) = r\sin(\omega t)$

b. $\phi = 5\pi/2$



- x-component $= r\cos\left(\omega t + \frac{5\pi}{2}\right)$
  $= r\cos(\omega t)\cos\left(\frac{5\pi}{2}\right) - r\sin(\omega t)\sin\left(\frac{5\pi}{2}\right)$
  $= -r\sin(\omega t)$
- y-component $= r\sin(\omega t + 5\pi/2)$
  $= r\sin(\omega t)\cos\left(\frac{5\pi}{2}\right) + r\cos(\omega t)\sin\left(\frac{5\pi}{2}\right)$
  $= r\cos(\omega t)$

c. $\phi = 135° = \frac{3\pi}{4}$ rad



• x-component $= r\cos\left(\omega t + \frac{3}{4}\pi\right)$

$$= r\cos(\omega t)\cos\left(\frac{3\pi}{4}\right) - r\sin(\omega t)\sin\left(\frac{3\pi}{4}\right)$$

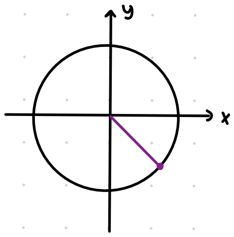
$$= -\frac{\sqrt{2}}{2} r\cos(\omega t) + \frac{\sqrt{2}}{2} r\sin(\omega t)$$

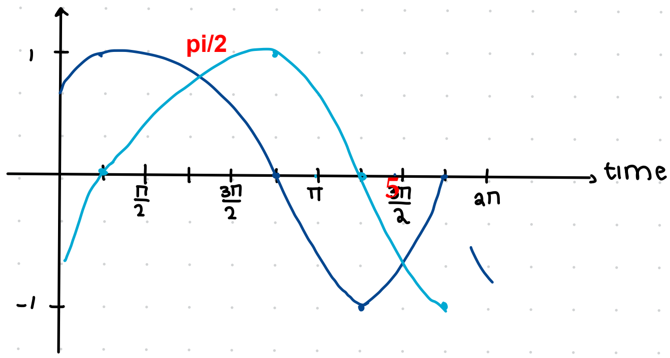
• y-component $= r\sin\left(\omega t + \frac{3\pi}{4}\right)$

$$= r\sin(\omega t)\cos\left(\frac{3\pi}{4}\right) + r\cos(\omega t)\sin\left(\frac{3\pi}{4}\right)$$

$$= -\frac{\sqrt{2}}{2} r\sin(\omega t) + \frac{\sqrt{2}}{2} r\cos(\omega t)$$

d $7\pi/4$ radians $= -45°$





• x-component $= r\cos\left(\omega t - \frac{\pi}{4}\right)$

$$= r\cos(\omega t)\cos\left(\frac{-\pi}{4}\right) - r\sin(\omega t)\sin\left(\frac{-\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{2} r\cos(\omega t) + \frac{\sqrt{2}}{2} r\sin(\omega t)$$

• y component $= r\sin\left(\omega t - \frac{\pi}{4}\right)$

$$= r\sin(\omega t)\cos\left(\frac{-\pi}{4}\right) + r\cos(\omega t)\sin\left(\frac{-\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{2} r\sin(\omega t) - \frac{\sqrt{2}}{2} r\cos(\omega t)$$

Problem 2

$\omega = 2\pi\nu$

$= 2\pi \cdot 12$

$= 24\pi$ [rad $\cdot s^{-1}$] unit conversion

≈ 75.4 rad$\cdot s^{-1}$

Anders Eggen
Siolene Croome
Arthur Humery
Ipek Incesu
Ryans Chen

CH-314 Structural Analysis
ruben.rodriguezmadrid@epfl.ch
ray.cowen@epfl.ch

Jigsaw 1E

Notations:
X Axis
Y Axis

Keeler Section 2.5. Frequency, oscillations and rotations

1. Sketch a graph of the x and y components of a rotating vector as a function of time for each of the given starting phases. Assume the phases are relative to the positive x-axis. In each case, comment on the form of your graphs, noting particularly whether they are simple sine or cosine functions.

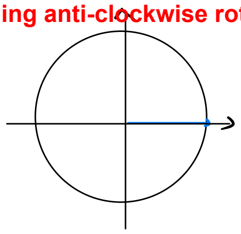
You are assuming anti-clockwise rotation...

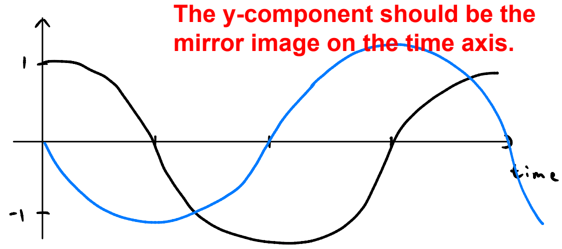
The y-component should be the mirror image on the time axis.

For all graphs: Assuming $r = 1$ ⇒ Unit trigonometric circle

a. 0°

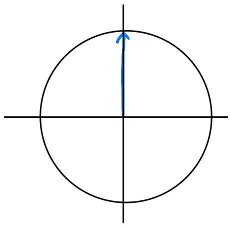
$x = \cos(\omega t)$
$y = \sin(\omega t)$

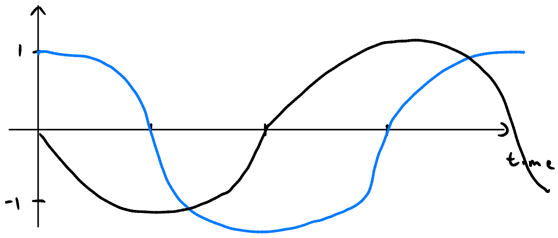




b. 5π/2 radians

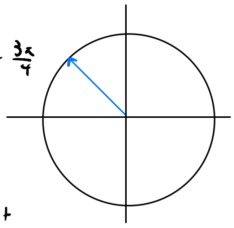
$\phi = \frac{5\pi}{2} = 2\pi + \frac{\pi}{2}$
$x(t) = \cos(\omega t + \frac{\pi}{2})$
$= -\sin(\omega t)$
$y(t) = \sin(\omega t + \frac{\pi}{2})$
$= \cos(\omega t)$

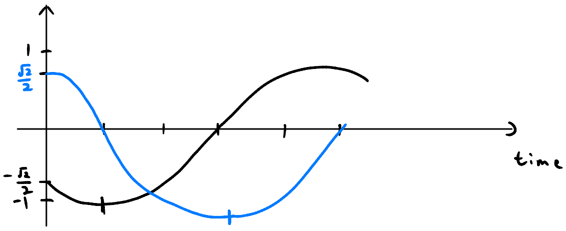




c. 135° $= \phi = \frac{3\pi}{4}$

$x = \cos(\omega t + \frac{3\pi}{4})$
$y = \sin(\omega t + \frac{3\pi}{4})$
⇒ $\frac{3\pi}{4}$ phase shift

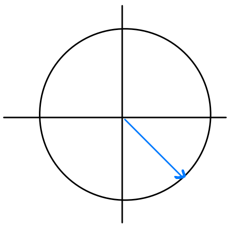




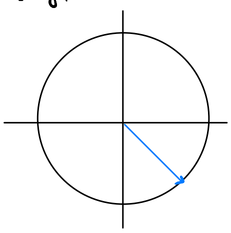
Same vector

d. 7π/4 radians

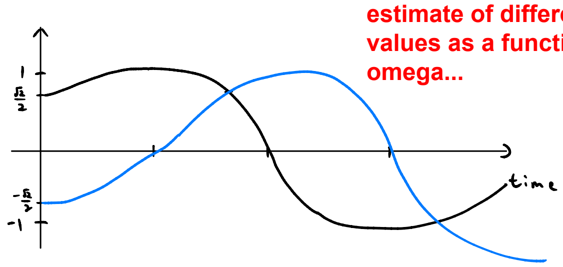
$x = \cos(\omega t - \frac{\pi}{4})$
$y = \sin(\omega t - \frac{\pi}{4})$
$-\frac{\pi}{4}$ phase shift



e. −45°



You could give an estimate of different t values as a function of omega...



2. Two peaks are separated by 12 Hz. What is this in rad s^{-1}?

$\Delta\omega = 2\pi f$, where $f = 12\,Hz$

$\Delta\omega = 2\pi \cdot 12 = 75{,}4\ rad \cdot s^{-1}$